

# Solution to the Liar and the Truth Teller

By Philip Thomas, January 2011

---

The solution to the original problem is "If I were to ask the other guard which door goes to heaven, what would he say?" If you ask the liar, he will lie about what the truth-teller would say, and point you to the door that goes to hell. The truth teller would tell the truth about how the liar would give the wrong door, and would also point you toward the door to hell. Thus, you take the other door.

For the version with only one guard, we ask "If I were to ask *you* which door goes to heaven, what would you say?". The truth teller would tell the truth and point you to the door to heaven. The liar would lie about what he *would* say if you asked him. If you did ask him, he would tell you the door to hell, so he lies about this also also points you toward the door to heaven. Thus both would point you toward the correct door. So, you don't need two guards, you only need one guard that either always lies or always tells the truth! This is also the solution to the problem with two guards, where you don't know how many tell the truth and how many lie - you just ignore one of them.

For the last variant, I will not prove that it is impossible to find who is who in the truth/liar/random problem with only three questions. However, I will answer the last part - can you figure out which is the door to heaven and hell in the truth/liar/random problem? Yes. Call the guards  $a$ ,  $b$  and  $c$ . Ask person  $a$  "If I were to ask you whether person  $b$  answers randomly, what would you say?". If the answer is *yes*, let  $x = c$ , otherwise let  $x = b$ . Notice that if  $a$  is the liar or the truth teller, he will answer truthfully and you know that  $x$  is not the guard who answers randomly. If  $a$  answers randomly, then you do not know whether or not his answer is correct... but you know that  $x$  is not the random-guard, as the random guard is  $a$ . Thus, you have found, in one question, a guard  $x$  whom you know is either a liar or a truth teller. You then ask  $x$  "If I were to ask you which door goes to heaven, what would you say?". He will tell you the correct door (see previous paragraph).

Notice that questions can be asked in the liar/truth-teller/random version for which there is no correct answer. You can handle these however you like - the answer provided will work regardless.